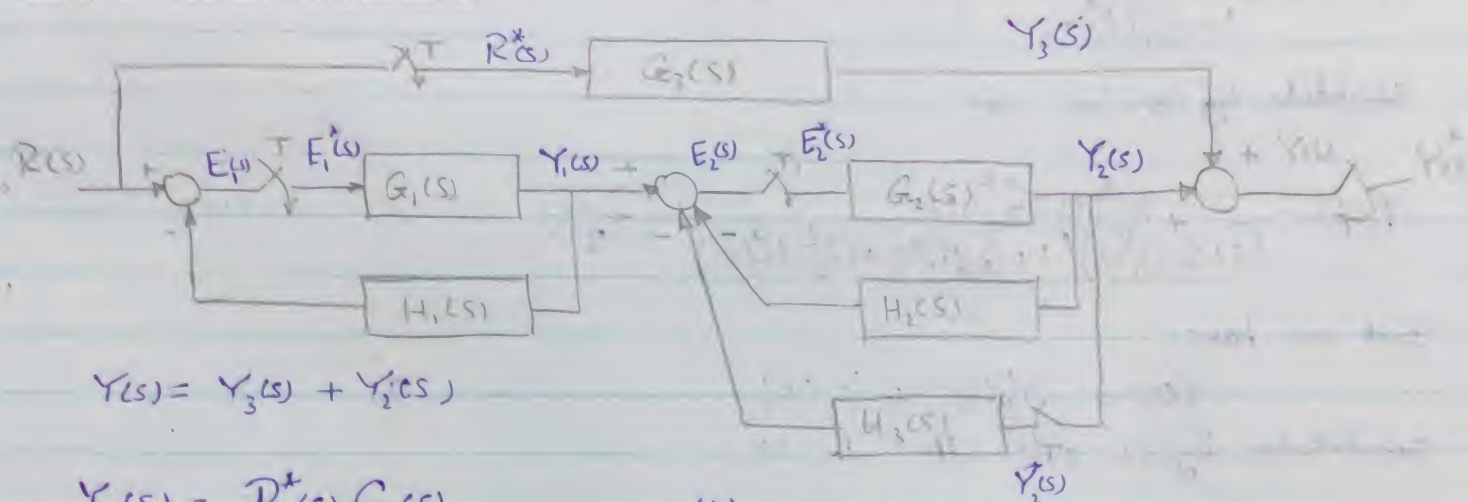


Digital Control Midterm Exam

Model Answer

Problem no. 1) part (a)



$$Y(s) = Y_3(s) + Y_2(s)$$

$$Y_3(s) = R^*(s) G_3(s) \quad \text{--- (1)}$$

$$E_1(s) = R(s) - H_1 Y_1(s) \quad \text{--- (2)}$$

$$Y_1(s) = E_1^*(s) G_1(s) \quad \text{--- (3)}$$

Substitute by (3) in (2)

$$E_1(s) = R(s) - G_1 H_1(s) E_1^*(s) \quad \text{--- (4)}$$

solving of (4)

$$E_1^*(s) = R^*(s) - G_1 H_1(s) E_1^*(s)$$

$$\therefore E_1^*(s) = \frac{R^*(s)}{1 + G_1 H_1(s)} \quad \text{--- (5)}$$

substitute by (5) in (3)

$$Y_1(s) = \frac{R^*(s) G_1(s)}{1 + G_1 H_1(s)} \quad \text{--- (6)}$$

$$E_2(s) = Y_1(s) - H_2 Y_2(s) - H_3 Y_3(s) \quad \text{--- (7)}$$

$$Y_2(s) = E_2^*(s) G_2(s) \quad \text{--- (8)}$$

Substitute by (6) and (8) in (7)

$$E_2(s) = \frac{R^*(s) G_1(s)}{1 + G_1 H_1(s)} - G_2 H_2(s) E_2^*(s) - G_2^*(s) H_3(s) E_2^*(s) \quad \text{--- (9)}$$

Starting of (9)

$$E_2^*(s) = \frac{R^*(s) G_1^*(s)}{1 + G_1 H_1^*(s)} - G_2 H_2^*(s) E_2^*(s) - G_2^*(s) H_3^*(s) E_2^*(s)$$

$$E_2^*(s) = \frac{R^*(s) G_1^*(s)}{1 + G_1 H_1^*(s)} \bigg/ (1 + G_2 H_2^*(s) + G_2^*(s) H_3^*(s)) \quad \dots (10)$$

Substitute by (10) in (8)

$$Y_2(s) = \frac{R^*(s) G_1^*(s)}{(1 + G_1 H_1^*(s))(1 + G_2 H_2^*(s) + G_2^*(s) H_3^*(s))} G_2(s) \quad \dots (11)$$

But we have

$$Y(s) = Y_2(s) + Y_3(s) \quad \dots (12)$$

Substitute by (1) and (11) in 12

$$\therefore Y(s) = \frac{R^*(s) G_1^*(s) G_2(s)}{(1 + G_1 H_1^*(s))(1 + G_2 H_2^*(s) + G_2^*(s) H_3^*(s))} + R^*(s) G_3(s) \quad \dots (13)$$

Starting of (13)

$$Y^*(s) = \frac{R^*(s) G_1^*(s) G_2^*(s)}{(1 + G_1 H_1^*(s))(1 + G_2 H_2^*(s) + G_2^*(s) H_3^*(s))} + R^*(s) G_3^*(s)$$

$$\therefore \frac{Y^*(s)}{R^*(s)} = \frac{G_1^*(s) G_2^*(s)}{(1 + G_1 H_1^*(s))(1 + G_2 H_2^*(s) + G_2^*(s) H_3^*(s))} + G_3^*(s) \quad \dots (14)$$

from (14) The closed loop digital transfer function is:

$$\boxed{\frac{Y(z)}{R(z)} = \frac{G_1(z) G_2(z)}{(1 + G_1 H_1(z))(1 + G_2 H_2(z) + G_2^*(z) H_3(z))} + G_3(z)}$$

Problem no (2) part (b)

i) c/c eqn $Z^3 - Z^2 + 0.25 = 0$

Solution using Jury test:

① $F(1) = (1)^3 - (1)^2 + 0.25 = 0.25 > 0$ (Satisfied)

② $(-1)^3 F(-1) = (-1)^3((-1)^3 - (-1)^2 + 0.25) = 1.75 > 0$ (Satisfied)

③ $|a_0| = 0.25 < |a_n| = 1$ Satisfied

④ Construct Jury array

$$\begin{array}{c|cccc} 0.25 & 0 & -1 & 1 \\ 1 & -1 & 0 & 0.25 \\ \hline b_0 & b_1 & b_2 & \end{array} \quad b_0 = \begin{vmatrix} 0.25 & 1 \\ 1 & 0.25 \end{vmatrix} = -0.9375$$

$$b_2 = \begin{vmatrix} 0.25 & 0 \\ 1 & -1 \end{vmatrix} = -0.25$$

$|b_0| = 0.9375 > |b_2| = 0.25$ (Satisfied)

The System is Stable

Solution using Bilinear Transformation:

for c/c eqn $Z^3 - Z^2 + 0.25 = 0$ let $Z = \frac{1+r}{1-r}$

$\left(\frac{1+r}{1-r}\right)^3 - \left(\frac{1+r}{1-r}\right)^2 + 0.25 = 0$

$(1+r)^3 - (1+r)^2(1-r) + 0.25(1-r)^3 = 0$

$1 + 3r + 3r^2 + r^3 - (1 + r - r^2 - r^3) + 0.25(1 + 3r + 3r^2 - r^3) = 0$

$0.25 + 1.25r + 4.75r^2 + 1.75r^3 \rightarrow$ Construct Routh Array

$$\begin{array}{l|ll} s^3 & 1.75 & 1.25 \\ s^2 & 4.75 & 0.25 \\ s^1 & 1.16 & \\ s^0 & 0.25 & \end{array}$$

There is no change in the Sign of 1st Column elements \rightarrow

The System is Stable

24) c/c eqn $Z^3 - 1.4Z^2 + 0.35Z + 0.05 = 0$

Solution using Jury test

① $F(1) = (1)^3 - 1.4(1)^2 + 0.35(1) + 0.05 = 0$ Not Satisfied (May Be Critical)

② $(-1)^3 F(-1) = (-1)^3((-1)^3 - 1.4(-1)^2 + 0.35(-1) + 0.05) = 2.7 > 0$ Satisfied

③ $|P_n| = 0.05 < |Q_n| = 1$ Satisfied

④ Construct Jury array

0.05	0.35	-1.4	1
1	-1.4	0.35	0.05
<hr/>			
b_0	b_1	b_2	

$b_0 = \begin{vmatrix} 0.05 & 1 \\ 1 & 0.05 \end{vmatrix} = -0.9975$

$b_2 = \begin{vmatrix} 0.05 & 0.35 \\ 1 & -1.4 \end{vmatrix} = -0.42$

$|b_0| = 0.9975 > |b_2| = 0.42$ Satisfied

The System is Critically Stable

Solution using Bilinear transformation $Z = \frac{1+r}{1-r}$

c/c eqn: $\left(\frac{1+r}{1-r}\right)^3 - 1.4\left(\frac{1+r}{1-r}\right)^2 + 0.35\left(\frac{1+r}{1-r}\right) + 0.05 = 0$

$(1+r)^3 - 1.4(1+r)^2(1-r) + 0.35(1+r)(1-r)^2 + 0.05(1-r)^3 = 0$

$(1+3r+3r^2+r^3) - 1.4(1+r-r^2-r^3) + 0.35(1-r-r^2+r^3) + 0.05(1-3r+3r^2-r^3) = 0$

$2.7r^3 + 4.2r^2 + 1.1r = 0$

Construct Routh array

r^3	2.7	1.1
r^2	4.2	
r^1	1.1	
r^0	0.1	

Aux eqn: $P(r) = 1.1r$

$\frac{dP(r)}{dr} = 1.1$

Solution of $P(r) = 0 \rightarrow r = 0$

one pole on z axis

The System is Critically Stable

Problem no (2)

1- $n_p = 2 \rightarrow z = 0, 1$
 $n_z = 2 \rightarrow z = 1.5, 2$

2- Real parts

To the left of odd number of poles and zeros

3- Asymptotes: Not found $n_p = n_z$

4- Breaking Points $k = \frac{-z(z-1)}{(z-1.5)(z-2)}$
 Breaking out (Max k)

z	0.5	0.55	0.6	0.65	0.70	0.75
k	0.166	0.179	0.19	0.198	0.201	0.2

Breaking Out at $z = 0.7$

Breaking in (Min k)

z	1.51	1.55	1.6	1.65	1.7	1.75
k	157	378	24	20.4	19.8	21

Breaking in at $z = 1.7$

5- No Departure angles

6- The root locus is a Circle with diameter $= 1.7 - 0.7 = 1 \rightarrow r = 0.5$
 and its center at $1.7 - 0.5 = 1.2$

7- Range of k for stability

$$k_{cr} = \frac{lp_1 lp_2}{l_z l_z} = \frac{1 \times 0.4}{1.3 \times 0.7} = 0.51$$

$$0 < k < 0.51$$

k_{cr} occurs at intersection between two circles

$$x^2 + y^2 = 1, (x-1.2)^2 + y^2 = 0.25$$

$$(x-1.2)^2 + (1-x^2) = 0.25 \rightarrow x = 0.9125$$

$$\therefore y = 0.41 \rightarrow k_{cr} \text{ at } z = 0.9125 + j0.41$$

$$k_{cr} = \frac{|z||z-1|}{|z-1.5||z-2|} = 0.504 \approx 0.51$$